Propagation of Acoustic Waves in a Two-Phase Vaporizing Mixture

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Nomenclature

a, a_0	=	speeds of sound in the droplet–gas mixture and in the pure gas
C_l, C_p	=	specific heats of liquid and gas
d	=	droplet diameter
h	=	enthalpy per unit mass
K	=	complex wave number, $k_1 + ik_2$
k	=	gas thermal conductivity
L	=	length of the domain
l_v	=	latent heat of vaporization
N_p	=	droplet number per unit volume of the mixture
Pr	=	Prandtl number
p	=	pressure of the mixture
T	=	temperature
t	=	time
u	=	velocity component in the x direction
α, β	=	spatial coefficients of attenuation and dispersion of acoustic energy
γ	=	specific heat ratio of gas
κ	=	mass ratio of liquid to gas per unit volume of the mixture (loading)
μ	=	gas dynamic viscosity
ρ	=	mass concentration (equal to density for the gas)
ρ_l	=	droplet density
τ_u, τ_t, τ_m	=	dynamic, thermal, and mass exchange response times in the Stokes regime
ω	=	angular frequency

Subscripts

p	=	dispersed phase
0	=	undisturbed state

Superscript

$'$	=	fluctuating component

Introduction

DROPLET vaporization and combustion have been identified as some of the major sources of instability in liquid-fueled propulsion systems.[1] The works of Braithwaite et al.[2] and more recently of Brooks and Beckstead[3] tend to prove that aluminum vaporization and combustion are also sources of instability for solid rocket motors. Because the aluminum droplets, due to their low volatility, burn across an extended area, this source might be significant on the global acoustic balance of a motor. In doing so, the condensed phase would then be at the origin of two antagonist processes because the damping arising from the velocity and thermal lags between the gas and inert particules or droplets has been known for more than 80 years.[4]

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Because the damping and driving mechanisms do not generally compensate themselves, reliable stability predictions must include these two contributions and, thus, must include the description of the reactive two-phase flow in the motor. The development of a numerical tool for the prediction of oscillation frequencies and levels in a motor is a quite challenging task that requires carreful and progressive validation steps. Inert particulate flows were first considered. Simulations were compared with the global acoustic balance theory for a cylindrical motor[5] and with experimental data recording on a small naturally unstable motor.[6,7] Because the comparisons were judged satisfactory in terms of frequency and level, it was decided to investigate reactive particulate flows. In this work, only pure vaporization is considered, first from a theoretical viewpoint by studying the interaction between an acoustic field and a vaporizing droplet cloud and then from a numerical viewpoint by using these results to test a numerical code developed for stability predictions.

Theoretical Development

Following Temkin and Dobbins's works,[8] limited to the case of an inert dispersed phase, Marble[9] then Davidson[10] studied sound attenuation in a condensing vapor. However, the distinction between the inert gas and the condensing vapor led them to write a mass equation for the vapor in addition to the mass equation for the inert gas and the dispersed phase. Because the objective of this study is to define a simple test case for validating at first single-species codes, the development will be quite different. Only the problem of the propagation of a plane and small-amplitude acoustic wave through a two-phase pure substance that may undergo phase exchange by vaporization will be considered here. Thus, the mass exchange process will be no longer a diffusion-controlled process but rather a thermal conductivity-controlled process. To simplify the problem, the expression of the mass transfer rate in a quasi steady state is used. The droplets are supposed to be rigid (no surface tension effects), spherical, and of uniform temperature and size (the regression of the droplet surface is negligible during one period of oscillation). For N_p droplets per unit volume of the mixture, the total mass transfer term is then

$$\dot{\omega} = \varepsilon N_p 2\pi d(k/C_p)\ln[1 + C_p(T - T_p)/l_v] \tag{1'}$$

where ε is either 1 (vaporization) or 0 to recover the results of Temkin and Dobbins.[8] The two-phase medium is assumed to be initially at a saturated state so that only the oscillatory motion will be at the origin of the nonequilibrium between both phases.

Following previous works,[8–10] a continuumlike description of the droplet cloud is retained. For a single class of droplets and assuming that the volume occupied by the liquid is negligible, the governing equations in a nonconservative form can be written as

$$\frac{\partial \rho}{\partial t} + \rho\frac{\partial u}{\partial x} + u\frac{\partial \rho}{\partial x} = \dot{\omega} \tag{2a}$$

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x}\right) + \frac{\partial p}{\partial x} = -F_D + \dot{\omega}(u_p - u) \tag{2b}$$

$$\rho C_p\left(\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x}\right) - \frac{\partial p}{\partial t} - u\frac{\partial p}{\partial x} = -F_D(u_p - u) - Q_v + \dot{\omega}\left[h(T_p) - \frac{p}{\rho_l} - h(T) + \frac{1}{2}(u_p - u)^2\right] \tag{2c}$$

for the gas phase and

$$\frac{\partial \rho_p}{\partial t} + \rho_p\frac{\partial u_p}{\partial x} + u_p\frac{\partial \rho_p}{\partial x} = -\dot{\omega} \tag{3a}$$

$$\rho_p\left(\frac{\partial u_p}{\partial t} + u_p\frac{\partial u_p}{\partial x}\right) = F_D \tag{3b}$$

$$\rho_p C_l\left(\frac{\partial T_p}{\partial t} + u_p\frac{\partial T_p}{\partial x}\right) = Q_v - \dot{\omega} l_v \tag{3c}$$

for the dispersed phase.

Because we are concerned only about fuel or metal droplets for which the gas-to-liquid density ratio is far lower than 1, the enthalpy of the liquid is equivalent to the internal energy. For the same reason, the history, added mass, and pressure gradient terms in the momentum equation can be neglected whatever the frequency. In these equations, it is also assumed that the mass efflux from the droplet has the velocity of the liquid phase, that the vapor is calorically perfect, and that gas dissipation terms are negligible with respect to two-phase dissipation terms. Finally, it is assumed that gas and droplet properties are unchanged, especially during one period of oscillation.

The total drag force F_D and the total heat transferred from the vapor to the liquid phase Q_v are written to highlight some characteristic times of exchange processes between both phases and by assuming that drag and heat transfer laws are unchanged by mass transfer. In the Stokes regime, that is a relevant assumption because the oscillatory motion has a small amplitude, it is possible to define

$$\tau_u = \rho_l d^2 \big/ (18\mu), \qquad \tau_t = 1.5(C_l/C_p) Pr\, \tau_u \tag{4}$$

so that

$$F_D = F'_D = N_{p0} 3\pi \mu d(u' - u'_p) = \rho_{p0}(u' - u'_p)/\tau_u \tag{5a}$$

$$Q_v = Q'_v = N_{p0} 2\pi k d(T' - T'_p) = \rho_{p0} C_l (T' - T'_p)/\tau_l \tag{5b}$$

Similarly, the total mass transfer term can be written as

$$\dot{\omega} = \dot{\omega}' = \frac{\varepsilon \rho_{p0} C_p (T' - T'_p)}{\tau_m l_v} \quad \text{with} \quad \tau_m = \frac{d^2 C_p \rho_l}{12k} = 1.5 Pr\, \tau_u \tag{6}$$

With these expressions, the linearized equations are

$$\frac{\partial}{\partial t}\left(\frac{\rho'}{\rho_0}\right) + a_0 \frac{\partial}{\partial x}\left(\frac{u'}{a_0}\right) = \frac{\varepsilon\kappa}{\lambda \tau_m}\left(\frac{T'}{T_0} - \frac{T'_p}{T_0}\right) \tag{7a}$$

$$\frac{\partial}{\partial t}\left(\frac{u'}{a_0}\right) + \frac{a_0}{\gamma}\frac{\partial}{\partial x}\left(\frac{\rho'}{\rho_0}\right) + \frac{a_0}{\gamma}\frac{\partial}{\partial x}\left(\frac{T'}{T_0}\right) = -\frac{\kappa}{\tau_u}\left(\frac{u'}{a_0} - \frac{u'_p}{a_0}\right) \tag{7b}$$

$$\frac{\partial}{\partial t}\left(\frac{T'}{T_0}\right) - (\gamma - 1)\frac{\partial}{\partial t}\left(\frac{\rho'}{\rho_0}\right) = -\gamma\kappa\left(\frac{C_l}{C_p}\frac{1}{\tau_t} - \frac{\phi}{\lambda}\frac{\varepsilon}{\tau_m}\right)\left(\frac{T'}{T_0} - \frac{T'_p}{T_0}\right) \tag{7c}$$

$$\frac{\partial}{\partial t}\left(\frac{u'_p}{a_0}\right) = \frac{1}{\tau_u}\left(\frac{u'}{a_0} - \frac{u'_p}{a_0}\right) \tag{7d}$$

$$\frac{\partial}{\partial t}\left(\frac{T'_p}{T_0}\right) = \left(\frac{1}{\tau_t} - \frac{C_p}{C_l}\frac{\varepsilon}{\tau_m}\right)\left(\frac{T'}{T_0} - \frac{T'_p}{T_0}\right) \tag{7e}$$

where, for convenience, we introduce

$$\lambda = \frac{l_v}{C_p T_0}, \qquad \phi = \frac{-(\gamma - 1)}{\gamma(\rho_0/\rho_l)} \tag{8}$$

The continuity equation of the dispersed phase is not used because the mass concentrations of both phases are linearly dependent through the loading, and κ is a constant parameter because of the equilibrium assumption.

A solution for the harmonic regime is obtained by writing that ρ', u', T', u'_p, and T'_p depend on x and t through a factor $\exp[i(Kx - \omega t)]$, where $K = k_1 + ik_2$ is unknown. The resulting homogeneous system has a nontrivial solution only if the determinant vanishes. Expanding the determinant and factoring the expression results in the following equation for K:

$$\left(\frac{K a_0}{\omega}\right)^2 \left[1 + \kappa \frac{C_l}{C_p}\frac{1 - \varepsilon(\phi/\lambda)}{1 - \varepsilon - i\omega\tau_t}\right] = \left\{1 + \gamma\kappa \frac{C_l}{C_p}\frac{1 - \varepsilon/\lambda[\phi + (\gamma - 1)/\gamma]}{1 - \varepsilon - i\omega\tau_t}\right\}\left(1 + \frac{\kappa}{1 - i\omega\tau_u}\right) \tag{9}$$

It is straightforward to recover the equation obtained by Temkin and Dobbins[8] by setting $\varepsilon = 0$. The resolution of this equation in term of k_1 and k_2 leads to closed-form expressions for the spatial coefficients of attenuation and dispersion of acoustic energy, $\alpha = 2k_2$ and $\beta = k_1^2 - (\omega/a_0)^2$.

Results and Comparison with Numerical Simulation

The variations of α and β vs the Stokes number $\omega\tau_u$ and for some values of λ and κ are shown in Fig. 1. The results are compared with Temkin and Dobbins's theory.[8] For vaporization, the acoustic wave propagation is strongly perturbed. With the adopted exchange model, the mass transfer amplifies the oscillation levels because the attenuation coefficient α becomes negative for some values of the Stokes number, whereas, without mass transfer, α is always positive. Concurrently, the propagation speed is increased for some values of the Stokes number ($\beta < 0$), whereas inert particles tend to slow it. The amplification effect and the speed of sound in the mixture, especially the value at equilibrium ($\omega\tau_u \to 0$), are strongly dependent on λ.

The vaporization process is intrinsically a driving mechanism, and its effect dominates the other physical (damping) processes because the α curve remains below the Temkin and Dobbins curve[8] whatever λ. The position of the minimum that gives an estimation of the critical frequency–diameter pair is dependent on κ and far less on λ. For a given λ, the position is shifted to large Stokes numbers, that is, to high frequencies and/or to large diameters, as κ increases. In contrast, the amplification rate seems to be less sensitive to this parameter. In other words, even a small mass fraction of vaporizing droplets can amplify acoustic waves.

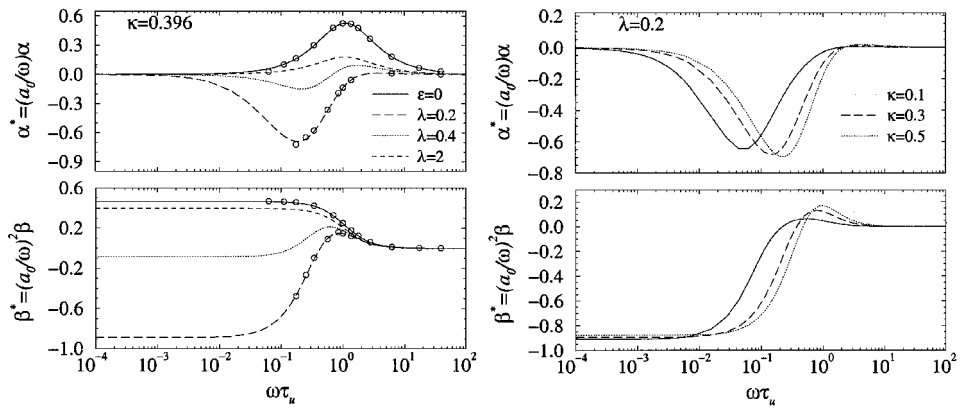

Fig. 1 Attenuation and dispersion for pure vaporization; parametric studies on λ and κ; circles correspond to numerical simulations ($\varepsilon = 0$, 1 and $\lambda = 0.2$).

The objective is now to numerically recover these results. A description of the code may be found in Refs. 5 and 6. The numeric is based on a time-splitting procedure. Each phase is first integrated using a finite volume explicit MacCormack scheme. Then, a third-order total variation diminishing Runge–Kutta time discretization is applied to update interphase exchange terms. In fact, because the medium is at rest, this test case is particularly selective only for the integration schemes for the source terms.

Characteristics of the test case described in Ref. 11 are retained. A sinusoidal pressure signal at 1000 Hz is imposed at the head end. The relative amplitude is chosen sufficiently small, namely 0.1%, to remain in the Stokes regime. The length of the domain corresponds to 20 wavelengths. Because the dispersion can be negative, computations are performed, at most, for only 15 periods to avoid spurious reflexions at the outlet. The solutions are computed with 50 points per wavelength on a uniform grid. Computations are run for several droplet diameters included between 5 and 75 μm to cover a large range of Stokes numbers. Gas and droplet properties are listed in Table 1.

Circles in Fig. 1 correspond to simulations performed for $\varepsilon = 0$ and 1. Figure 2 gives an example of pressure amplification for 6-μm vaporizing droplets. As shown in Fig. 1, the third-order explicit scheme is not robust enough to recover theoretical results below a Stokes number of 0.15, whereas, without mass transfer ($\varepsilon = 0$), the scheme gives good results down to a Stokes number of 0.06. In fact, an implicit method seems to be required when the source terms become stiff. Except for these small diameters, numerical results are close to the theoretical attenuation and dispersion, thereby validating the code. Simulations with different λ and κ were not judged necessary. Note that the droplet diameter and loading remain constant during the computation (to the fourth digit), so that all of the computations are relevant.

Table 1 Gas and droplet properties

Property	Value	Property	Value
p_0	5 MPa	κ	0.396
ρ_0	3.78 kg/m^3	ρ_l	1766 kg/m^3
C_p	2021.8 J/kg/K	C_l	0.68 C_p
γ	1.23	λ	0.2
μ	8.855 10^{-5} kg/m/s	Pr	0.8

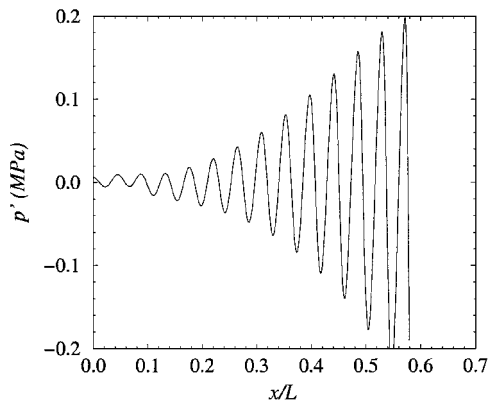

Fig. 2 Amplification effect for 6-μm vaporizing droplets ($\lambda = 0.2$ and $\omega\tau_u = 0.25$).

Conclusions

In spite of its simplicity, this test case seems to be valuable to obtain a first validation of an unsteady reactive two-phase flow code. Beyond this numerical aspect, this study highlights the driving effect for acoustic-related oscillations of a thermal conductivity-controlled vaporization process and incidentally gives an estimation of the critical frequency–diameter pairs in a straightforward way. The next step in the validation of numerical codes is to consider a diffusion-controlled vaporization process to address multi-species aspects.

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Erratum

THE November–December 2001 issue of the *Journal of Propulsion and Power* contained two errors. In the editorial preface on page 1137, we misspelled Woody Waesche. On the spine, we misspelled Hypersonics.
AIAA regrets both errors.